

THE USE OF THE PLANE WAVE FLUID-STRUCTURE
INTERACTION LOADING APPROXIMATION IN NASTRAN

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ABSTRACT

The Plane Wave Approximation (PWA) is widely used in finite element analysis to implement the loading generated by an underwater shockwave. The method required to implement the PWA in NASTRAN is presented along with example problems. A theoretical background is provided and the limitations of the PWA are discussed.

INTRODUCTION

Background

The finite element method is commonly used for analysis of structures exposed to underwater shockwaves. Modeling the structure using the finite element method is of less concern than loading the structure with a shockwave in a fluid medium. The fluid and the structure interact and the loading will be modified by the structure moving in the water. If the structure moves faster than the fluid can respond, then the density of the water near the structure will diminish considerably, causing, in simplified terms, a void in the water. This condition is known as cavitation and will cause the loading from the shockwave to be abated. As the structure slows, the fluid density will return to normal, or the void in the water will collapse due to ambient water pressure. Water closure, sometimes called a water hammer effect, occurs when the cavitation void closes and water slams together causing a momentum transfer. If this happens before the shockwave has passed the structure, then the shockwave loading will continue.

In order to implement the underwater explosion shockwave loading, many finite element codes employ a load function known as the Plane Wave Approximation (PWA). The chief advantage of the PWA is that it is much less complex than other methods which employ modeling the fluid itself. This simplicity translates into faster computer run times with less memory requirements.

Consider a water bounded node of a finite element model. Let:

y = displacement of the node

t = time

$P_i = P_0 e^{-t/\theta}$ be the incident shockwave

$P_r = \phi(t)$ be the reflected pulse off the model

ρ_a = mass per unit area associated with each node

ρ_0 = density of water

θ = time constant of pressure pulse (assumed exponential)

A = water surface area associated with each node

The equation of motion of the node is:

$$\rho_a \ddot{y} = P_0 e^{-\frac{t}{\theta}} + \phi(t) \quad (1)$$

Continuity at the surface of the node stipulates that:

$$\rho_0 c \dot{y} = P_0 e^{-\frac{t}{\theta}} - \phi(t) \quad (2)$$

Eliminating ϕ using equations (1) and (2) results in:

$$\rho_a \ddot{y} + \rho_0 c \dot{y} = 2P_0 e^{-\frac{t}{\theta}} \quad (3)$$

The resulting force applied at the node is given by:

$$F(t) = A \rho_a \ddot{y} = \left[2P_0 e^{-\frac{t}{\theta}} - \rho_0 c \dot{y} \right] A \quad (4)$$

The above equations were originally derived by Taylor in 1941 (reference 1). The solution is exact as long as the water does not cavitate. The water may cavitate at the node or a short distance away from the node. Cavitation at the node will happen when:

$$\rho_0 c \dot{y} \geq 2P_0 e^{-\frac{t}{\theta}} \quad (5)$$

The nodal force derived from the PWA can be stated by combining equations (4) and (5).

$$F(t) = \begin{cases} 0 & ; \rho_0 c \dot{y} \geq 2P_0 e^{-\frac{t}{\theta}} \\ \left[2P_0 e^{-\frac{t}{\theta}} - \rho_0 c \dot{y} \right] A & ; \rho_0 c \dot{y} < 2P_0 e^{-\frac{t}{\theta}} \end{cases} \quad (6)$$

Objective

The objective of this paper is to demonstrate the use of the PWA for underwater shockwave loading in COSMIC NASTRAN for transient analysis problems.

IMPLEMENTATION OF PLANE WAVE APPROXIMATION IN NASTRAN

The PWA can be implemented in COSMIC NASTRAN without the use of the DMAP option. The procedure is facilitated by the use of scalar points and extra points, as these points will add neither mass nor stiffness to the finite element model. Consider a scalar dashpot which is implemented by the CDAMP4 card. The dashpot is constrained to ground by leaving one of the coordinates on the CDAMP4 card blank. This will allow the dashpot to function independently of the structure which is being analyzed. The dashpot is given the value of $\rho_0 c$ and loaded with an exponential shockwave:

$$P(t) = 2P_0 e^{-\frac{t}{\theta}} \quad (7)$$

Equation (7) is best implemented with the TLOAD2 card. NASTRAN will not handle an instantaneous acceleration; therefore, equation (7) must be ramped initially to its peak value. The ramp is established with the TLOAD1 card coupled with the TABLED1 card. A ramp of 10 time steps to peak value was chosen using engineering judgment. The additional 10 time steps increased execution time only marginally while allowing sufficient ramp time to use the TLOAD2 card. Therefore, no attempt was made to fine tune the number of time steps in the ramp further. The TLOAD2 card is made to start on the 10th time step by using the DELAY card. The TLOAD1 and TLOAD2 card are executed together with the DLOAD card as shown in figure 1. From first principles, the velocity of the scalar point is:

$$\dot{u}(t)_{\text{scalar pt.}} = \frac{2P_0 e^{-\frac{t}{\theta}}}{\rho_0 c} \quad (8)$$

The wet node, extra point, and scalar point are equated using the TF card, as shown in figure 2. Each water bounded node must be assigned its own unique scalar dashpot, extra point and TF card. The TF relation defines the extra point velocity:

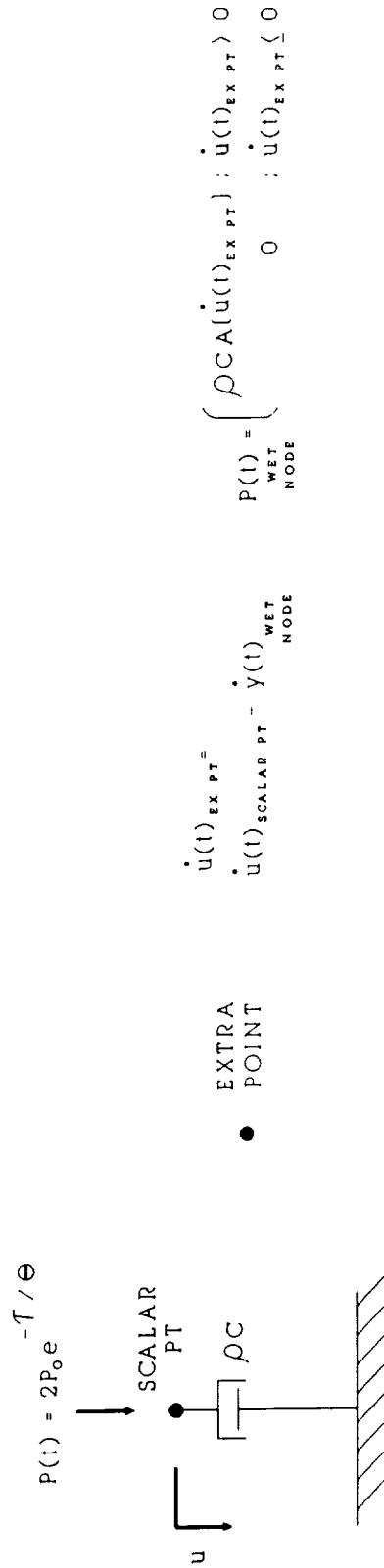
$$\dot{u}(t)_{\text{ex. pt.}} = \dot{u}(t)_{\text{scalar pt.}} - \dot{y}(t)_{\text{wet node}} \quad (9)$$

The NONLIN3 card is used to actually load the finite element model. Each wet node must have its own unique NONLIN3 card and corresponding TF card. As shown in figure 2, the NONLIN3 relation is:

$$F(t)_{\text{wet node}} = \begin{cases} \rho_0 c A [\dot{u}(t)_{\text{ex. pt.}}] ; \dot{u}(t)_{\text{ex. pt.}} > 0 \\ 0 ; \dot{u}(t)_{\text{ex. pt.}} \leq 0 \end{cases} \quad (10)$$

By using equations (8), (9), and (10) the PWA will result (see equation (6)).

$$F(t)_{\text{wet node}} = \begin{cases} A \left[2P_0 e^{-\frac{t}{\theta}} - \rho_0 c \dot{y}(t) \right] ; \dot{u}(t)_{\text{ex. pt.}} > 0 \\ 0 ; \dot{u}(t)_{\text{ex. pt.}} \leq 0 \end{cases} \quad (11)$$



CDAMP4	EPOINT	TF	NONLIN3
ONE SCALAR POINT DAMPER FOR EACH WET NODE	ONE EXTRA POINT FOR EACH WET NODE	APPLY CONSTRAINT EQUATION FOR EACH WET NODE	APPLY AT ALL WET NODES

Fig. 2. - Schematic of PWA

ILLUSTRATIVE PROBLEMS

Piston Problem

Consider the one dimensional problem of a rigid piston in a cylinder with a constant pressure on one side and an exponential shockwave in a fluid medium on the other (figure 3). The water can cavitate at the piston or at some distance away from the surface of the piston. The piston problem was solved in closed form by Gordon and Handleton in 1985 (reference 2) and is categorized by the term:

$$\lambda = \frac{\rho_a}{\rho_o c \theta} \quad (12)$$

The piston can be modeled in NASTRAN with a single plate element constrained to move only in one direction. Four scalar dashpots are used for the four nodes of the plate element, and the dashpots are loaded with an exponential shockwave using the parameters shown in figure 3. The procedure described above is utilized to employ the PWA in NASTRAN. The results are shown in figure 4 in terms of displacement vs. time for varying values of λ . NASTRAN was executed until the point of maximum displacement occurred. For $\lambda = 5.0$ no cavitation occurs, and the PWA solution offers excellent agreement with the closed form solution of Gordon.

When $\lambda = 0.5$ cavitation occurs at the plate, the PWA differs from the closed form solution in terms of maximum displacement by approximately 36%. This difference happens after 15 msec. Prior to this the PWA in NASTRAN has good agreement with the closed form solution. The difference could be attributed to the fact that the closed form solution has the additional impulse due to water closure. The PWA does not model the closure event.

At $\lambda = 0.05$ the PWA in NASTRAN and the closed form solution of Gordon differ by 156% in terms of peak displacement. Cavitation occurs away from the surface of the piston, in the water itself. The PWA in NASTRAN does not indicate any cavitation as it only tracks the water edge at the piston. The PWA and the closed form begin to separate at approximately 4 msec, much earlier than when $\lambda = 0.5$. Again, much of the difference could be attributed to the lack of a water hammer effect in the PWA.

It is apparent that the accuracy of utilizing the PWA for the piston problem is dependent upon the value of λ . Practically speaking, $\rho_o c$ remains fairly constant and θ can only be varied over a small range. The term which can allow the most deviation is the mass of the piston ρ_a . Thus, the PWA works well for a large mass term which does not allow cavitation. As the mass decreases, the effects of cavitation become more pronounced and the accuracy of the PWA decreases dramatically.

Somewhere between $\lambda = 0.5$ and 5.0 is the crossover value of λ , where no difference exists between the solution of Gordon and the PWA. Assuming a linear relationship exists between λ and the peak displacement difference between Gordon and PWA, the crossover can be found. Accordingly, for

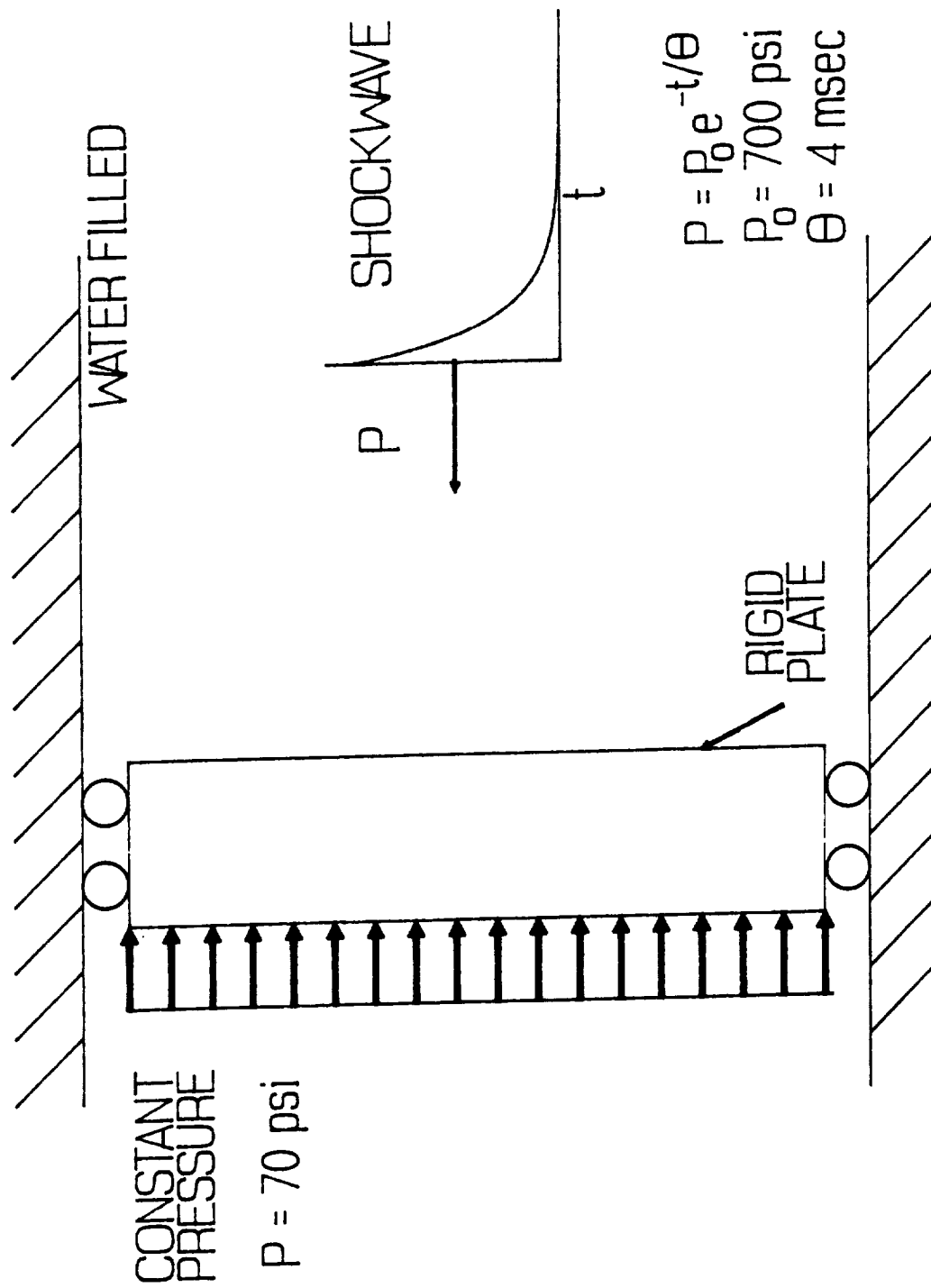


Fig. 3. The Piston Problem

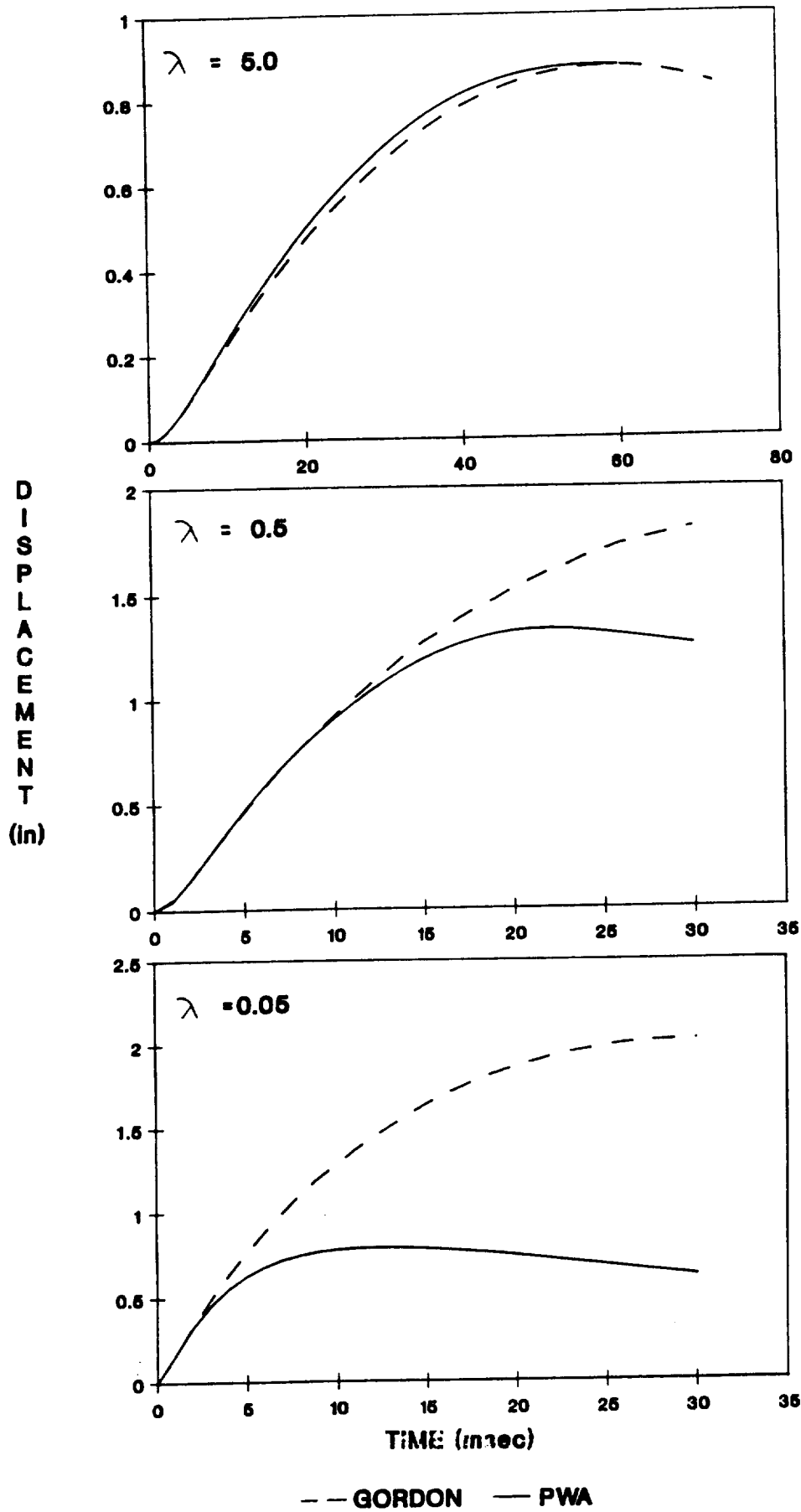


Fig. 4. - Comparison of Results from Piston Problem

$\lambda = 0.64$ and greater, no difference in displacement should exist between PWA and the closed form solution of Gordon.

Circular Flat Plate With Mass

In this example a 2-inch thick flat circular plate is welded to a rigid annular backing structure. The backing structure constrains the outside of the plate but allows the inner section of the plate to move. In the center of the plate a dummy mass is welded which weighs approximately the same as the plate. The plate has air on one side, water on the other. A short distance from the plate is an explosive charge which will release an exponential shockwave when detonated.

The experiment was conducted by the Underwater Explosions Research Division of the David Taylor Research Center and is shown schematically in figure 5. Velocity meters were placed on the mass to record the experimental response.

A finite element model was formulated of the plate and mass using COSMIC NASTRAN. Using symmetry, only a quarter of the plate was modeled using plate elements. The mass was formulated using triangular plate elements and the plate with quadrilateral elements. The plate was modeled as pin connected at the edge. The mesh used is shown in figure 6. The PWA as described above was used to load the model.

In this case the lowest value of λ is 0.84. Although this problem is very different than the piston problem, if the problems were analogous we would expect no cavitation and the PWA to be an excellent method in which to employ the underwater shockwave loading. The results from the NASTRAN finite element model are plotted versus the experimental velocity in figure 6. The analytical solution offers good agreement with experimental results in terms of initial average acceleration, average deceleration and peak velocity. These results support the use of the PWA for this application.

CONCLUDING REMARKS

The PWA is a useful engineering method for analyzing the shock response of naval vessels from underwater explosions. The applicability of the method is dependent upon the presence of cavitation in the water.

The PWA can be employed in NASTRAN for transient analysis problems without using the DMAP option. Standard bulk data deck cards can be used.

The method requires the calculation for each wet node of several parameters (A, P_0, θ) as well as an angular correction term if the shockwave and structure are not perpendicular. Additionally, several bulk data deck cards will be needed for each wet node. An accounting system must be set up in order to tie together the correct bulk data deck cards to the proper wet node. If the finite element model is of sufficient size then accomplishing

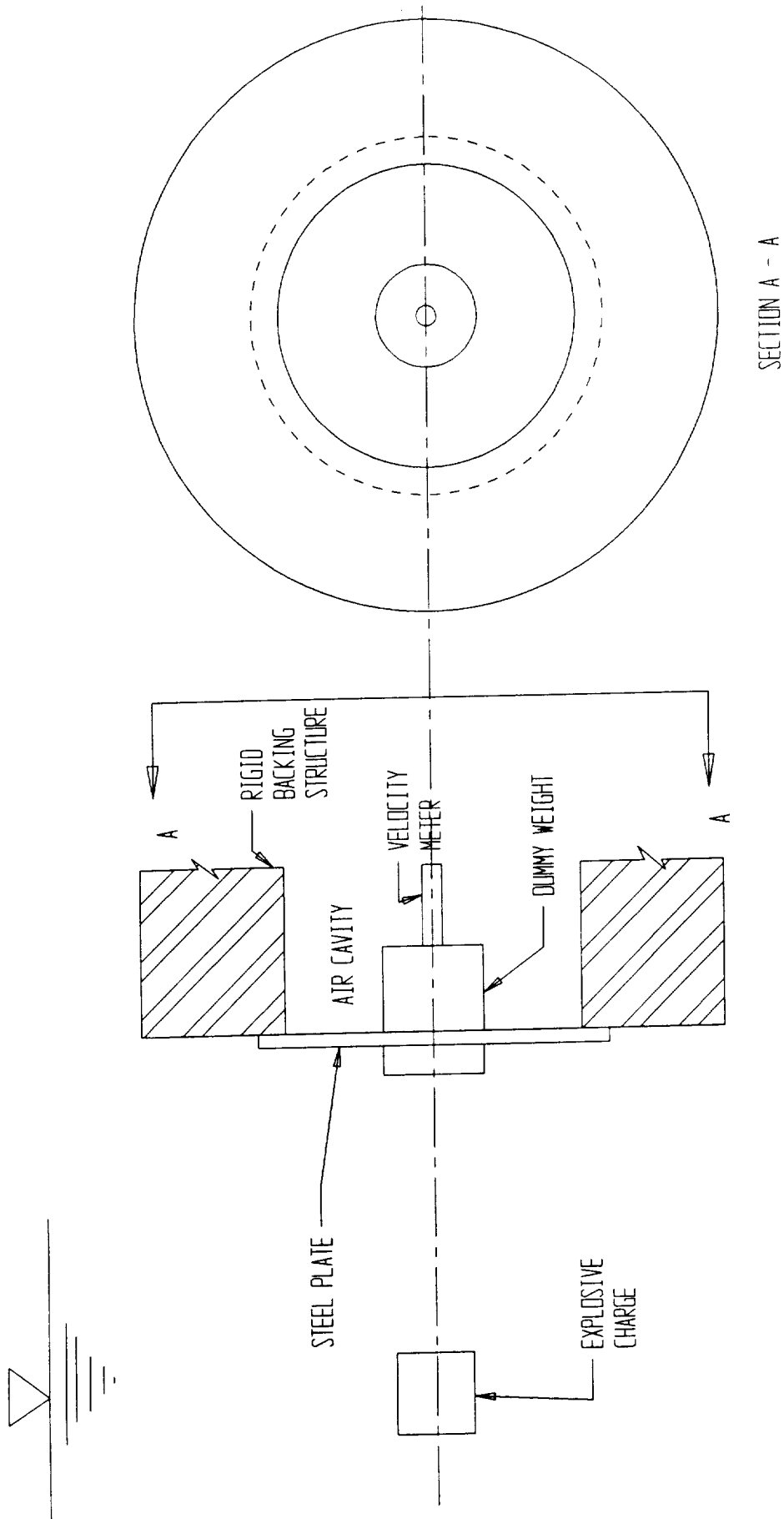


Fig. 5. - Test Arrangement

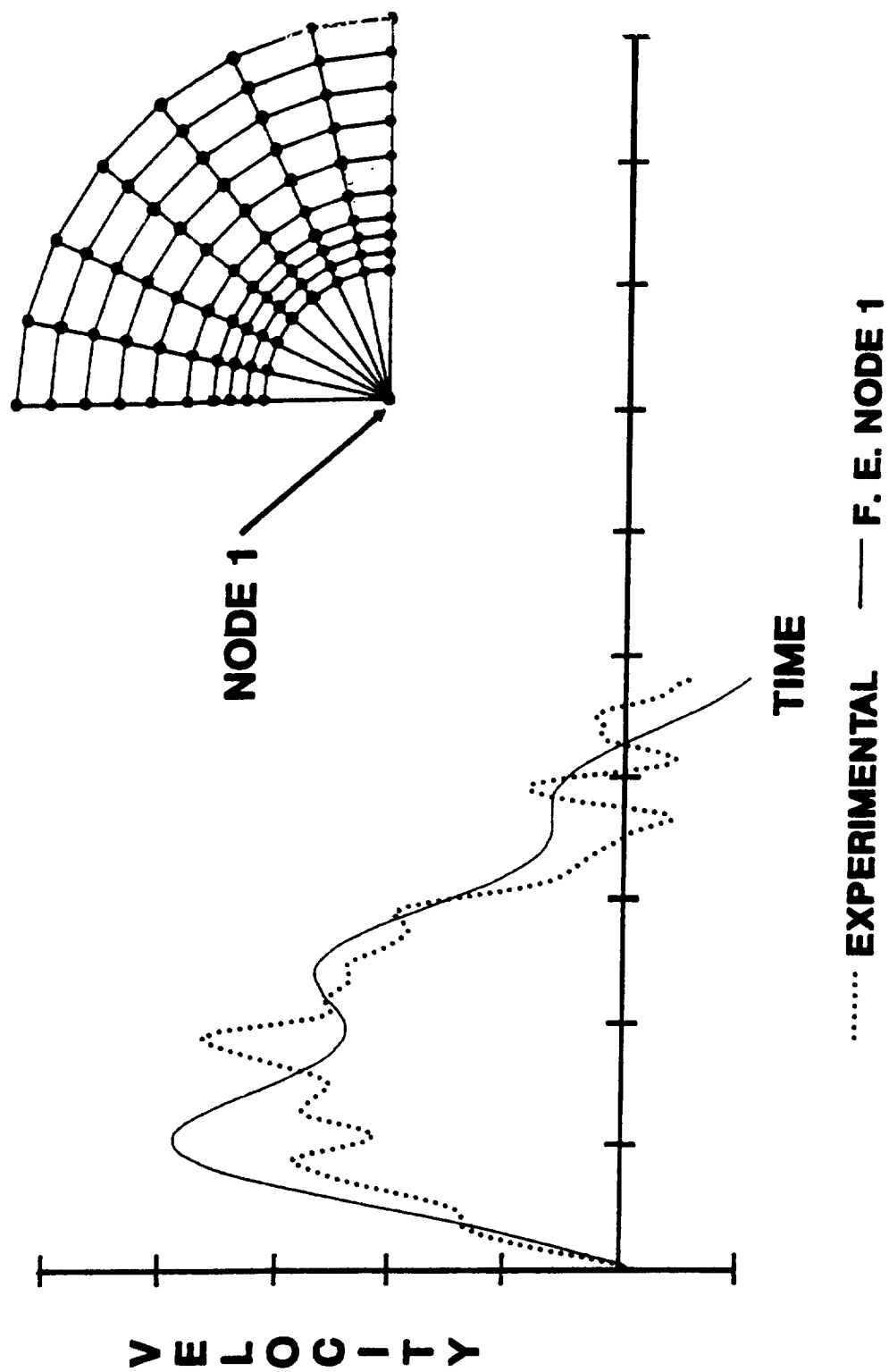


Fig. 6. - Comparison of Results from Experiment and P/A

the above can be quite a labor intensive task. A preprocessor program, which could scan the NASTRAN input deck and output the required bulk data deck cards necessary to implement the PWA, is recommended.

REFERENCES

1. Taylor, G. I., "The Pressure and Impulse of Submarine Explosion Waves on Plates," Underwater Explosion Research, volume 1, Office of Naval Research, 1950.
2. Handleton, R. and Gordon, J., "Energy Absorption at a Restrained Mass," David W. Taylor Naval Ship Research and Development Center report SD-85-24, March 1985.